Hand calculation of PI to 20 digits

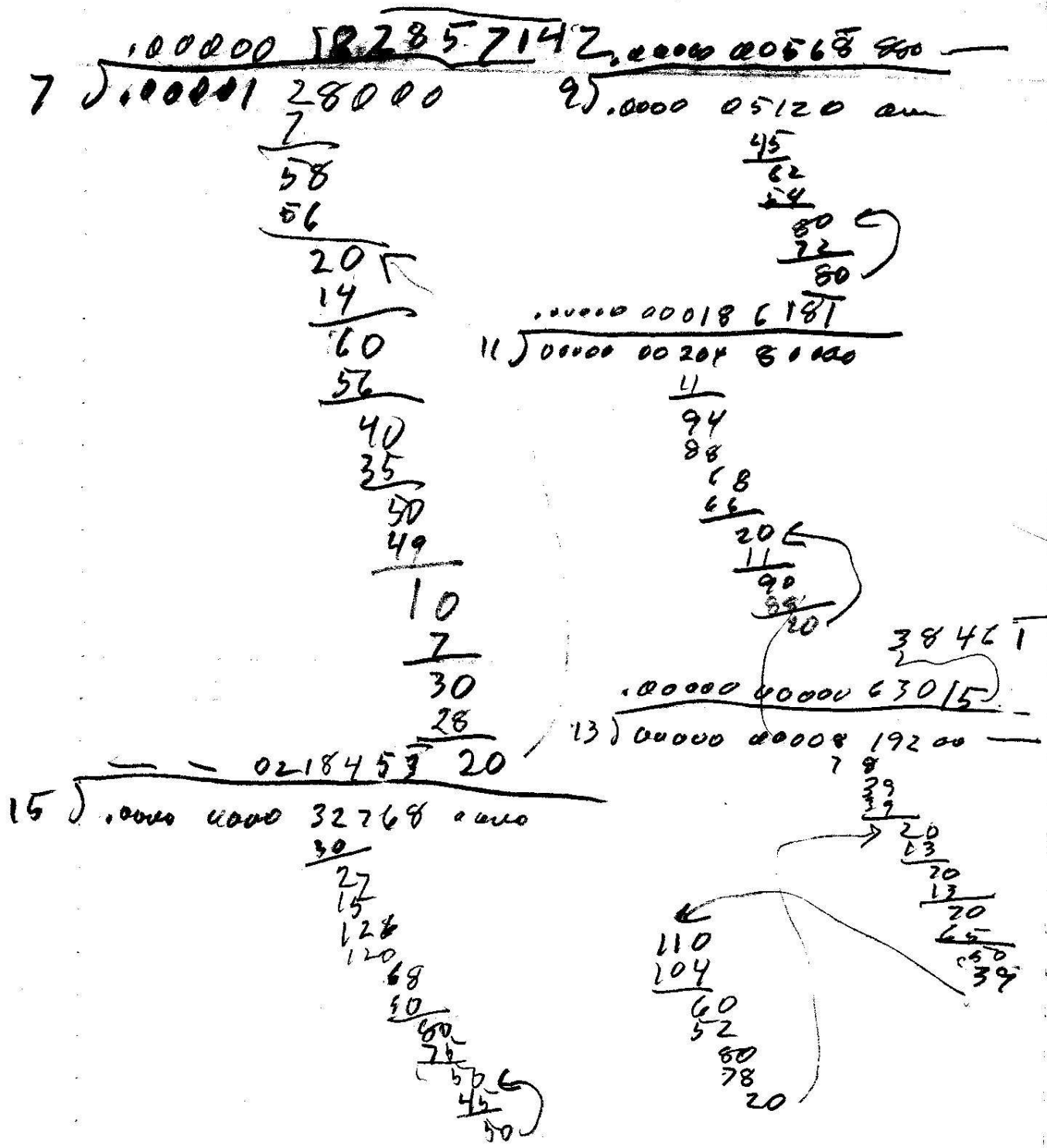
The following seven pages contain the hand calculation of PI to 20 digits of which there was an error of 14 units in the last digit, effected the last two digits and resulted in 18 correct digits. This is more than digits needed for most every day needs. The planet Pluto is on the average 6,000,000,000 km from the sun, this would be 6X10^15 mm, so this value of PI would give the circumference the nearest mm. This example has several problems first we do not know the distance to the mm and the orbit is not a circle or a perfect ellipse.

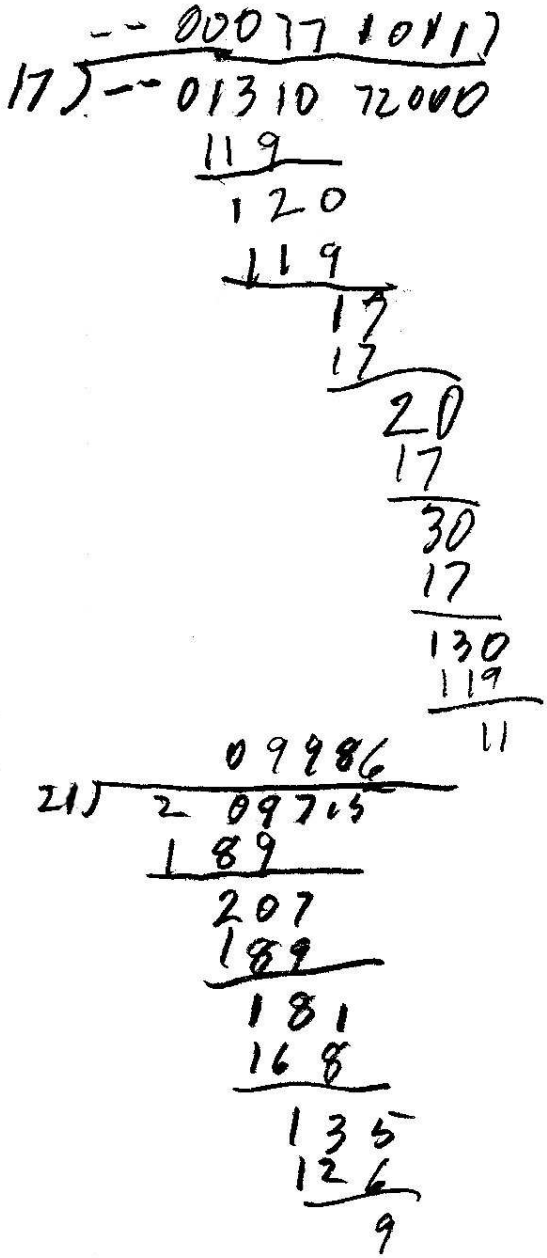
In 1706 John Machin discovered the following formula relation and used it to calculate 100 digits of PI. This formula has become one of the most used since it was discovered, PI = 4 * (4 * ATN(1/5) - ATN(1/239). The function ATN(X) = X - X^3/3 + X^5/5 – X^7/7 + X^9/9 etc. this relation can be used to calculated the ATN function. -

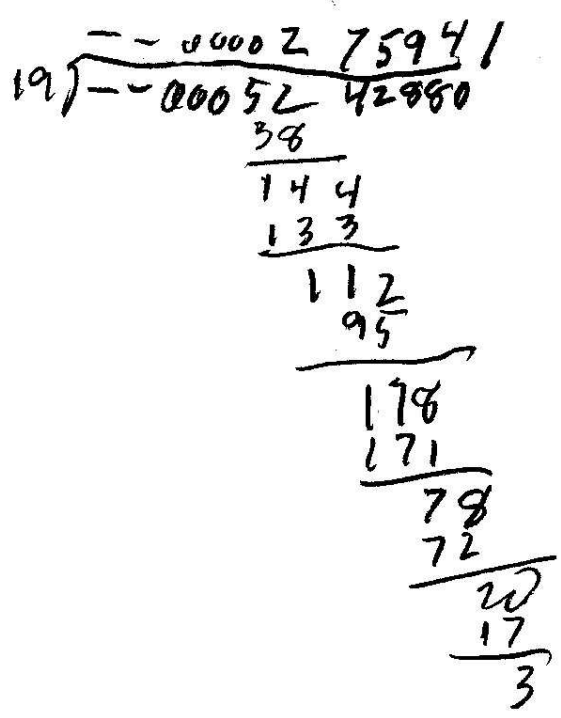
The first three pages are for the of the ATN(1/5). The forth page is for has the final calculations to produce the final value of PI. The last three are the calculation of ATN(1/239). In the left margin I produced a table of values 57121 and as a check the last value is 10 times so I would know that there were no errors.

I started the hand calculation doing 10 digits and then expanded it to this 20 digit example, which rook about 2 hours. To do the full 40 digit would take a full day. I have started to do the work on 40 digits with 4 10 digits group and I have found it to hard to keep the columns straight, I will have to start over with 5 digit groups. To equal the work of John Machin would take several weeks and a whole lot more paper. To do 100 digits the 1/5 has 70 terms while the 1/239 has 21 terms and each term has 5 times as many digits as the example here.

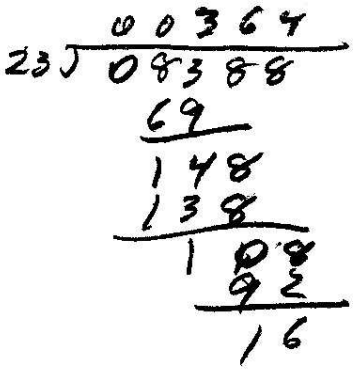
1	.200000	00000	00000	00000
3	.008000	00000	00000	00000
5	.000032	00000	00000	00000
7	.000001	28000	00000	00000
9	.000000	05120	00000	00000
11	.000000	00204	80000	00000
13	.000000	00008	19200	00000
15	.000000	00000	32768	00000
17	.000000	00000	01310	72000
19	.000000	00000	00052	42880
21	.000000	00000	00002	09712
23	.000000	00000	00000	08388
25	.000000	00000	00000	00335
3	.00266	66666	22222	66666
7	.000100	18285	71428	57142
11	.000001	00016	61818	18181
15	.000006	00000	02184	53333
19	.000000	00000	00002	75941
23	00000	00000	00000	00364
−	.00266	84971	02100	71627
1	.200000	00000	00000	00000
5	.000006	40006	00000	00000
9	.000060	00568	88888	88888
13	00000	00000	63015	38461
17	00000	00000	00077	10117
21	00000	00001	00000	09986
25	00000	00000	00000	00013
	.200016	40569	51991	47465
−	00266	84971	02100	71627
ATN(1/5)	19739	55598	49880	75838




--00077 10117
17)--01310 72000
119
120
119
17
17
20
17
30
17
130
119
11


--00025 94180
19)--00052 42880
38
144
133
112
95
178
171
78
72
20
17
3

09986
21)2 09765
189
207
189
181
168
135
126
9


00364
23)08388
69
148
138
108
92
16

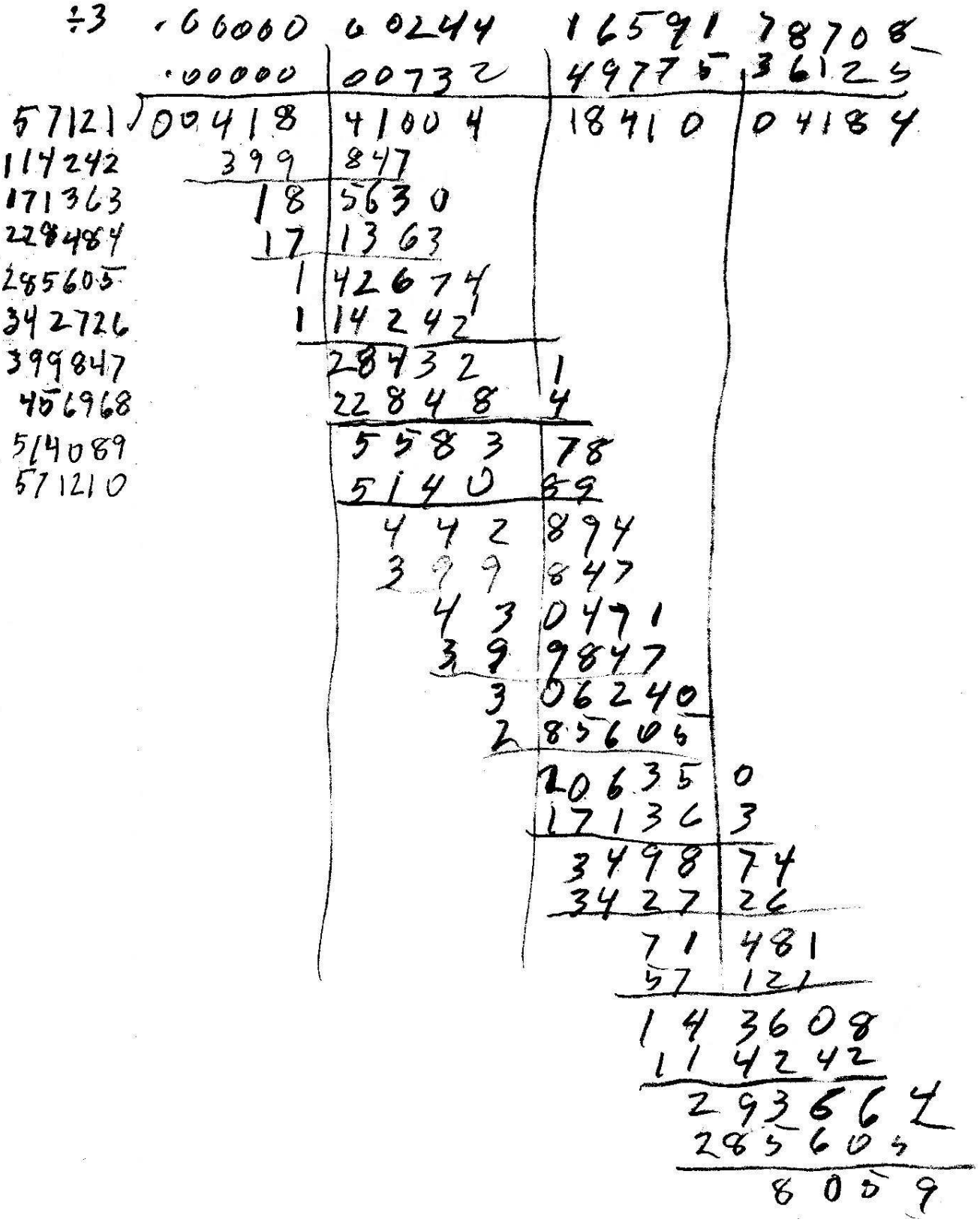
$ATN(\frac{1}{5})$	.19739	55598	49880	75838
$\times 4$				$\times 4$
$4 \times ATN(\frac{1}{5})$	.78958	22393	99523	03352
$-ATN(\frac{1}{239})$	.00418	40760	02074	72387
$\pi/4$	.78539	81633	97448	30965
$\times 4$	.			$\times 4$
π	3.14159	26535	89793	23860

$$
\begin{array}{r}
.00418\ 41 \\
239\overline{)1.00000\ \ 00000\ \ 00000\ \ 00000} \\
\underline{956} \\
440 \\
\underline{239} \\
2010 \\
\underline{1912} \\
980 \\
\underline{956} \\
240 \\
\underline{239} \\
1
\end{array}
$$

$$
\begin{array}{r}
239 \\
\times\ 239 \\
\hline
2151 \\
717 \\
478 \\
\hline
57121
\end{array}
$$

3	00000	00244	16591	78708
7				00320
	00000	00244	16591	79028
1	.00418	41004	18410	04184
5			00256	47231
+	.00418	41004	18666	51415
−		00244	16591	79028
ATN($\frac{1}{239}$)	00418	40760	02074	72387

1	00418	41004	18410	04184
3	00000	00732	49775	36125
5	00000	00000	01282	36157
7	00000	00000	00000	02244



÷6

00251 472314

00000 00000 01282 36157

57121) 00000 00732 49775 36125

571 21

161 287

114 242

47 0457

45 6968

1 34895

1 14242

-- 00000 02244   206533

57121)-- 01282 36125   171363

114242   351706

139941   342726

114342   89801

255992   57121

228484   326802

275085   285605

228484   411975

46601   399847

12128

0320

7) 2244